1. **Introduction**
   1. Remote sensing provides toolsets that allow for better understanding of our environment but the ways we answer scientific questions about changes through space and time need to be rigorous and robust.
   2. PARTS is a method of statistical inference for remotely sensed spatiotemporal datasets. This method was developed to answer ecological questions from these data in a statistical appropriate way – one that accounts for both spatial and temporal autocorrelation. PARTS can detect trends that other methods are poor at detecting and provides a toolset for map-scale hypothesis testing about the existence and drivers of temporal trends.
   3. PARTS is a two-part model in which temporal variation is collapsed and spatial autocorrelation among residuals is used in the mode’s covariance structure. [Description of the statistical model formulation]
   4. While PARTS is statistically valid and outperforms other methods, further investigation into its robustness under strenuous conditions is needed. To that end, we will conduct a series of simulation studies to evaluate the performance of PARTS under conditions designed to push its limits. We aim to:
      1. evaluate and compare the accuracy of PARTS when data are generated with either a gaussian or non-gaussian spatial error structure
      2. evaluate and compare Type I error under the gaussian and non-gaussian cases
      3. evaluate the accuracy of PARTS when data are generated with an unknown fixed spatial autocorrelation component
      4. evaluate the accuracy of PARTS to detect temporal trends with varying ranges of spatial autocorrelation
      5. evaluate the accuracy of PARTS to detect the effect of a spatiotemporal independent variable on temporal trends, with varying spatial and temporal autocorrelation.
      6. Compare PARTS to a ‘gold standard’ model (gplmm ARMA)
2. **Methods**
   1. We simulated 104x104 pixel map datasets to test the robustness of PARTS. The full simulation model used for data generation is X(t) = λL + t(βL)+ γW(t) + ψR + ε(t) where [describe model]. Due to computational demands, the number of simulations conducted to address each goal differed…
   2. 2 static land classes were established in a checkerboard pattern across the map…
   3. To address goals (i) and (ii), we generated spatial-only datasets by setting alpha, beta, gamma, and phi equal to 0 and by setting t equal to 1.
      1. Lambda0 was set to 0 and lambda1 to was set to 0.2. theta was set to one of 0, 0.05, or 0.25 and errors were either N(0,Sigma) or T[3]\*Sigma.
      2. 500 simulations were conducted for each case and, to allow for accurate estimates of type I errors in the non-gaussian case, an additional 500 were conducted under theta = 0 and e(1) = T[3]\*Sigma.
      3. We calculated lambda - lambdahat for each simulation and estimated Type I errors by calculating the proportion of significant tests (p < 0.05) in cases where lambda = 0.
   4. To address goal (iii), we also generated spatial-only datasets (as in c.) but with the errors following N(0, 0.5\*Sigma) and phi1 = 0.5 – this ensures that R and e(1) have comparable effects on X. R is generated with a 2D wave function [describe function] with component Txy defining the period.
      1. Data were generated with either Txy = 1 (one cycle per map), Txy = 4 (4 cycles per map), and Txy = 9 (nine cycles per map). These levels were chosen to provide variation in the overlap of waves with land classes.
      2. 200 simulations per case were conducted.
      3. We calculated lambda - lambdahat for each simulation
   5. To address goal (vi), we added temporal variation back into the model by setting t = 30 and beta1 = 1/30 to allow the effect of land class 1 to increase over time. Spatial autocorrelation was generated with the tapered covariance function [describe]. Temporal autocorrelation in e(t) was generated with AR parameter rho=0.4.
      1. The range of the spatial autocorrelation theta was set to one of 0, 0.05, or 0.25
      2. 200 simulations per case were conducted (N = 600).
      3. We calculated beta - betahat for each simulation
   6. To address goal (v), we added a spatiotemporal independent variable by setting gamma = 1 and generating W(t) in the same manner as e(t). In these simulations, we fixed the range of spatial autocorrelation in the error term to 0.25.
      1. We set the range of spatial autocorrelation in W to one of 0 or 0.25
      2. We set the AR parameter for W to one of 0 or 0.4.
      3. 200 simulations per case were conducted (N = 800).
      4. We calculated gamma - gammahat for each simulation.
   7. Finally, to address goal (vi), we fit PARTS and a pglmm ARMA model [decscribe model] to the same datasets for comparison. Due to the computational and optimization restrictions of pglmm ARMA, these simulations were structured differently.
      1. Simulated maps were 8x8 pixels with no separate land classes (i.e. L0 = L1)
      2. Data were simulated according to X(t)= λ + βt + ε(t) where lambda is the baseline (intercept) effect, beta is the effect of time t and spatial autocorrelation in e(t) is now generated by an exponential covariance function rather than the tapered function.
      3. Beta was set to one of 0, 0.25, 0.5, 0.75 (Power analysis)
      4. 1000 simulations were conducted (Type I error) for each case (N = 4000)
3. **Results**
   1. Simulations took a total of 497+24 hours on 4 3600 MHz cores.
   2. Accuracy was consistent across map size, but precision increased with larger maps.
   3. PARTS accurately estimated spatial parameter lambda at all levels in both gaussian and non-gaussian cases.
   4. The type I error rate of PARTS under both gaussian and non-gaussian conditions was 4.7%
   5. PARTS did not accurately estimate lambda with unmeasured fixed spatial variation.
      1. For Txy = 1, peaks aligned with land class 0 while valleys aligned with land class 1. Unsurprisingly, this led to overestimation of lambda0 and underestimation of lambda1.
      2. For Txy = 4, peaks aligned with land class 1 while valleys aligned with land class 0. Unsurprisingly, this led to underestimation of lambda0 and overestimation of lambda1.
      3. For Txy = 9, peaks and valleys were evenly distributed across land class 1 and land class 0. This resulted in the best estimates of lambda0 and lambda1 in this simulation set.
   6. PARTS accurately estimated beta across all values of theta
   7. PARTS accurately estimated gamma across all combinations of rho\_w and theta\_w
   8. PARTS and pglmm ARMA had similar power and type I errors. PARTS had only sightly higher Type I (.042 compared to .033) and power at a 0.05 significance threshold.
4. **Discussion** 
   1. PARTS is an accurate and robust method for detecting trends in spatiotemporal datasets.
      1. PARTS reports reliable statistical significance of effects: PARTS does not suffer from inflated Type I error nor loss of power – in fact, PARTS performs better than the ‘gold standard’ method in this respect.
      2. PARTS is accurate even on moderately-sized maps, though precision increases with size. This indicates that PARTS is suitable for real-world datasets of almost any size.
      3. PARTS is suitable for estimating 1) purely spatial effects (lambda), 2) time trend effects (beta), and 3) spatio-temporal effects (gamma)
      4. PARTS is insensitive to wide variation in the range of spatial autocorrelation (theta).
      5. PARTS even performs well when errors are non-gaussian – indicating that assumptions of normality are not highly important for this method.
   2. One caveat to this is that PARTS performs poorly when a fixed source of spatial variation is unaccounted for (i.e., R), especially when this source of variation is confounded with the independent variable of interest (i.e., land class). This is similar to the multi-collinearity problem in regression and researchers should cautiously consider possible sources of fixed variation that are not related to the variable of interest.